

Self-Consistent Solutions for Bulk Gravity-Matter Systems Coupled to Lightlike Branes

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Abstract

We study self-consistent $D=4$ gravity-matter systems coupled to a new class of Weyl-conformally invariant *lightlike* branes (*WILL*-branes). The latter serve as material and charged source for gravity and electromagnetism. Further, due to the natural coupling to a 3-index antisymmetric tensor gauge field, the *WILL*-brane dynamically produces a *space-varying* bulk cosmological constant. We find static spherically-symmetric solutions where the space-time consists of two regions with black-hole-type geometries separated by the *WILL*-brane which “straddles” their common event horizon and, therefore, provides an explicit dynamical realization of the “membrane paradigm” in black hole physics. Finally, by matching via *WILL*-brane of internal Schwarzschild-de-Sitter with external Reissner-Nordström-de-Sitter (or external Schwarzschild-de-Sitter) geometries we discover the emergence of a potential “well” for infalling test particles in the vicinity of the *WILL*-brane (the common horizon) with a minimum on the brane itself.

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I. INTRODUCTION

Higher-dimensional extended objects gained in recent years a dramatically increasing importance due to various developments in string theory, gravity, astrophysics and cosmology.

In non-perturbative string theory (for a background, see refs.[1]) there arise several types of higher-dimensional membranes (p -branes, Dp -branes) which play a crucial role in the description of string dualities, microscopic physics of black holes, gauge theory/gravity correspondence [2], large-radius compactifications of extra dimensions, cosmological brane-world scenarios [3], model building in high-energy particle phenomenology [4], *etc.*

Lightlike membranes are of particular interest in general relativity as they describe impulsive lightlike signals arising in various violent astrophysical events, *e.g.*, final explosion in cataclysmic processes such as supernovae and collision of neutron stars [5]. Lightlike membranes are basic ingredients in the so called “membrane paradigm” theory [6] which appears to be a quite effective treatment of the physics of a black hole horizon. Furthermore, the thin-wall description of domain walls coupled to gravity [7, 8] is able to provide neat models for many cosmological and astrophysical effects.

In refs.[7, 8] lightlike membranes in the context of gravity and cosmology have been extensively studied from a phenomenological point of view, *i.e.*, by introducing them without specifying the Lagrangian dynamics from which they may originate. Recently in a series of papers [10, 11] we have developed a new field-theoretic approach for a systematic description of the dynamics of lightlike branes starting from concise *Weyl-conformally invariant* actions. The latter are related to, but bear significant qualitative differences from, the standard Nambu-Goto-type p -brane actions[20] (here $(p + 1)$ is the dimension of the brane world-volume, see Sect.II below).

Furthermore, our Weyl-conformally invariant actions are *not* related to the the p -brane actions previously proposed in the literature [12] where the standard Weyl-conformally non-invariant Nambu-Goto p -brane actions and their supersymmetric counterparts were reformulated in a formally Weyl-invariant form by means of introducing auxiliary non-dynamical fields with a non-trivial transformation properties under Weyl-conformal symmetry appropriately tuned up to compensate for the Weyl non-invariance with respect to the original dynamical degrees of freedom. Namely, one immediately observes that the latter formally Weyl-invariant p -brane actions *do not change* the dynamical content of the standard Nambu-

Goto p -branes (describing inherently *massive* modes). This is in sharp contrast to the presently discussed Weyl-conformally invariant p -brane models, which describe *intrinsically lightlike* p -branes for any even p (i.e., for any odd-dimensional world-volume). In what follows we will use for the latter the acronym *WILL-branes* (Weyl-invariant lightlike branes).

Our approach is based on the general idea of employing alternative non-Riemannian integration measures (volume-forms) in the actions of generally-covariant (reparametrization-invariant) field theories instead of (or, more generally, on equal footing with) the standard Riemannian volume forms. Namely, instead of (or alongside with) the standard Riemannian integration measure density $\sqrt{-g}$ with $g = \det \|g_{\mu\nu}\|$ being the determinant of the corresponding Riemannian metric, one can employ the equally well suited non-Riemannian integration measure density:

$$\Phi(\varphi) \equiv \frac{1}{D!} \varepsilon_{i_1 \dots i_D} \varepsilon^{m_1 \dots m_D} \partial_{m_1} \varphi^{i_1} \dots \partial_{m_D} \varphi^{i_D} , \quad (1)$$

where φ^i ($i = 1, \dots, D$) denote auxiliary scalar fields. This idea has been first proposed and applied in the context of four-dimensional theories involving gravity [13] by introducing a new class of “two-measure” gravitational models. It has been demonstrated that the latter models are capable to provide plausible solutions for a broad array of basic problems in cosmology and particle physics, such as: (i) scale invariance and its dynamical breakdown; (ii) spontaneous generation of dimensionful fundamental scales; (iii) the cosmological constant problem; (iv) the problem of fermionic families; (v) applications to dark energy problem and modern cosmological brane-world scenarios. For a detailed discussion we refer to the series of papers [13, 14].

Subsequently, the idea of employing an alternative non-Riemannian integration measure was applied systematically to string, p -brane and Dp -brane models [15]. The main feature of these new classes of modified string/brane theories is the appearance of the pertinent string/brane tension as an additional dynamical degree of freedom beyond the usual string/brane physical degrees of freedom, instead of being introduced *ad hoc* as a dimensionful scale. The dynamical string/brane tension acquires the physical meaning of a world-sheet electric field strength (in the string case) or world-volume $(p+1)$ -form field strength (in the p -brane case) and obeys Maxwell (Yang-Mills) equations of motion or their higher-rank anti-symmetric tensor gauge field analogues, respectively. As a result of the latter property the modified-measure string model with dynamical tension yields a simple classical mechanism

of “color” charge confinement [15].

The above mentioned modified-measure p -brane and Dp -brane models [15] share the same drawback as ordinary Nambu-Goto p -branes, namely that Weyl-conformal invariance is lost beyond the simplest string case ($p = 1$). On the other hand, the form of the action of the modified-measure string model with dynamical tension suggested a natural way to construct explicitly the new class of *Weyl-conformally invariant* p -brane models for any p [10, 11].

The present paper has a two-fold objective. First, in Section II we briefly review (and extend) the construction of *WILL-branes*, including a Kalb-Ramond-type coupling of the latter to a space-time $(p + 1)$ -rank antisymmetric tensor gauge field. In the second main part we derive systematically spherically-symmetric solutions for the coupled system of bulk Einstein-Maxwell interacting with a *WILL-brane* (Sections III,IV), the latter serving as a matter and charged source and, in addition, producing a space-varying dynamical cosmological constant. Finally, in Section V we describe a physically interesting effect of creation of a potential “well” around the *WILL-brane*, which divides space-time as a common horizon into two separate regions with different black hole geometries and whose matching is explicitly given in terms of the free *WILL-brane* parameters (electric charge and Kalb-Rammond coupling constant).

II. WEYL-CONFORMALLY INVARIANT LIGHTLIKE BRANES

A. Action and Equations of Motion. Lightlike Property

Consider the following new kind of p -brane action involving *modified world-volume measure* $\Phi(\varphi)$ (cf. (1)) and an auxiliary (Abelian) world-volume gauge field A_a [10, 11]:

$$S = - \int d^{p+1}\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}} \right] \quad (2)$$

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \dots i_{p+1}} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} \varphi^{i_1} \dots \partial_{a_{p+1}} \varphi^{i_{p+1}} \quad (3)$$

Here γ_{ab} denotes the intrinsic Riemannian metric on the brane world-volume, $\gamma = \det \|\gamma_{ab}\|$, $F_{ab} = \partial_a A_b - \partial_b A_a$ and $a, b = 0, 1, \dots, p; i, j = 1, \dots, p + 1$.

The above action is invariant under Weyl (conformal) symmetry for any p :

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho \gamma_{ab} \quad , \quad \varphi^i \longrightarrow \varphi'^i = \varphi'^i(\varphi) \quad (4)$$

with Jacobian $\det\left\|\frac{\partial\varphi'^i}{\partial\varphi^j}\right\| = \rho$.

Rewriting the action (2) in the following equivalent form:

$$S = - \int d^{p+1}\sigma \chi \sqrt{-\gamma} \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab} F_{cd} \gamma^{ac} \gamma^{bd}} \right] \quad , \quad \chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \quad (5)$$

we see that the composite field χ plays the role of a dynamical (variable) brane tension. Let us note the following differences of (2) (or (5)) w.r.t. the standard Nambu-Goto p -branes (in the Polyakov-like formulation) :

- New non-Riemannian integration measure density $\Phi(\varphi)$ instead of the usual $\sqrt{-\gamma}$, and no “cosmological-constant” term $((p-1)\sqrt{-\gamma})$.
- Variable brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ which is Weyl-conformal *gauge dependent*: $\chi \rightarrow \rho^{\frac{1}{2}(1-p)} \chi$.
- Auxiliary world-sheet gauge field A_a in a “square-root” Maxwell term [21]. As discussed in our previous papers on the subject [10, 11], the appearance of this “square-root” Maxwell term is naturally required for consistency of the *WILL-brane* dynamics.
- Possibility for natural couplings of auxiliary A_a to external world-volume (“color” charge) currents J^a .
- Weyl-invariant for *any* p ; describes *intrinsically light-like* p -branes for any even p (i.e., odd-dimensional world-volume). Let us also note that there are *NO quantum conformal anomalies* in *odd* xdimensions!.

Employing the short-hand notations:

$$(\partial_a X \partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \quad , \quad \sqrt{FF\gamma\gamma} \equiv \sqrt{F_{ab} F_{cd} \gamma^{ac} \gamma^{bd}} \quad , \quad (6)$$

the equations of motion w.r.t. measure-building auxiliary scalars φ^i and γ^{ab} read, respectively:

$$\frac{1}{2} \gamma^{cd} (\partial_c X \partial_d X) - \sqrt{FF\gamma\gamma} = M \quad \left(= \text{const} \right) \quad (7)$$

$$\frac{1}{2} (\partial_a X \partial_b X) + \frac{F_{ac} \gamma^{cd} F_{db}}{\sqrt{FF\gamma\gamma}} = 0 \quad (8)$$

Taking the trace in (8) implies $M = 0$ in Eq.(7).

Next, we get the equations of motion w.r.t. auxiliary gauge field A_a :

$$\partial_b \left(\frac{F_{cd} \gamma^{ac} \gamma^{bd}}{\sqrt{F F \gamma \gamma}} \Phi(\varphi) \right) = 0 \quad (9)$$

and the equations of motion w.r.t. X^μ :

$$\partial_a (\Phi(\varphi) \gamma^{ab} \partial_b X^\mu) + \Phi(\varphi) \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu = 0 \quad (10)$$

where $\Gamma_{\nu\lambda}^\mu = \frac{1}{2} G^{\mu\kappa} (\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda})$ is the affine connection corresponding to the external space-time metric $G_{\mu\nu}$.

Consider the γ^{ab} -equations of motion (8); in fact, the latter are constraints analogous to the (classical) Virasoro constraints in ordinary string theory. Since F_{ab} is anti-symmetric $(p+1) \times (p+1)$ matrix, then F_{ab} is *not invertible* in any odd $(p+1)$ – it has at least one zero-eigenvalue vector-field V^a ($F_{ab} V^b = 0$). Therefore, for any odd $(p+1)$ the induced metric $(\partial_a X \partial_b X)$ on the world-volume of the Weyl-invariant brane is *singular* (as opposed to the ordinary Nambu-Goto brane (!)) :

$$(\partial_a X \partial_b X) V^b = 0 \quad , \quad \text{i.e.} \quad (\partial_V X \partial_V X) = 0 \quad , \quad (\partial_\perp X \partial_V X) = 0 \quad (11)$$

where $\partial_V \equiv V^a \partial_a$ and ∂_\perp are derivatives along the tangent vectors in the complement of V^a .

Thus, we arrive at the following important conclusion: every point on the world-surface of the Weyl-invariant p -brane (2) (for odd $(p+1)$) moves with the speed of light in a time-evolution along the zero-eigenvalue vector-field V^a of F_{ab} . Therefore, we will name (2) (for odd $(p+1)$) by the acronym *WILL-brane* (Weyl-Invariant Lightlike-brane) model.

Remark. In what follows we will use a natural ansatz for the world-volume electric field $F_{0i} = 0$ implying that $(V^a) = (1, \underline{0})$, i.e., $\partial_V = \partial_0 \equiv \partial_\tau$.

B. Special case $p = 2$: WILL-Membrane

Henceforth we will explicitly consider the special case $p = 2$ of (2), i.e., the Weyl-invariant lightlike membrane model:

$$S = - \int d^3\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \sqrt{F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}} \right] \quad (12)$$

$$\Phi(\varphi) \equiv \frac{1}{3!} \varepsilon_{ijk} \varepsilon^{abc} \partial_a \varphi^i \partial_b \varphi^j \partial_c \varphi^k \quad , \quad a, b, c = 0, 1, 2 \quad , \quad i, j, k = 1, 2, 3 \quad (13)$$

Invariance under world-volume reparametrizations allows to introduce the following standard (synchronous) gauge-fixing conditions:

$$\gamma^{0i} = 0 \quad (i = 1, 2) \quad , \quad \gamma^{00} = -1 . \quad (14)$$

The residual $\tau \equiv \sigma^0$ -independent reparametrization invariance allows for further conformally-flat gauge-fixing of the space-like part of γ_{ab} :

$$\gamma_{ij} = a(\tau, \sigma^1, \sigma^2) \tilde{\gamma}_{ij}(\sigma^1, \sigma^2) \quad (15)$$

with $\tilde{\gamma}_{ij}$ a standard reference $2D$ metric on the membrane surface.

The ansatz $F_{0i} = 0$ together with the gauge-fixed equations motion for A_a (9) implies:

$$\partial_i \chi = 0 , \quad (16)$$

where $\chi \equiv \frac{\Phi(\phi)}{\sqrt{-\gamma}}$ (the dynamical brane tension).

Employing (14), the remaining gauge-fixed equations of motion w.r.t. γ^{ab} and X^μ read (recall $(\partial_a X \partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$):

$$(\partial_0 X \partial_0 X) = 0 \quad , \quad (\partial_0 X \partial_i X) = 0 , \quad (17)$$

$$(\partial_i X \partial_j X) - \frac{1}{2} \gamma_{ij} \gamma^{kl} (\partial_k X \partial_l X) = 0 , \quad (18)$$

(the latter look exactly like the classical (Virasoro) constraints for an Euclidean string theory w.r.t. (σ^1, σ^2));

$$\square^{(3)} X^\mu + (-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl} \partial_k X^\nu \partial_l X^\lambda) \Gamma_{\nu\lambda}^\mu = 0 , \quad (19)$$

$$\square^{(3)} \equiv -\frac{1}{\chi \sqrt{\gamma^{(2)}}} \partial_0 \left(\chi \sqrt{\gamma^{(2)}} \partial_0 \right) + \frac{1}{\sqrt{\gamma^{(2)}}} \partial_i \left(\sqrt{\gamma^{(2)}} \gamma^{ij} \partial_j \right) . \quad (20)$$

C. Coupling to Maxwell and Rank-3 Antisymmetric Tensor Gauge Field

We can extend straightforwardly the *WILL*-brane model via couplings to external space-time electromagnetic field \mathcal{A}_μ and, furthermore, to external space-time rank 3 gauge potential $\mathcal{A}_{\mu\nu\lambda}$ (Kalb-Ramond-type coupling) keeping *manifest Weyl-invariance*:

$$\begin{aligned} S = & - \int d^3 \sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab} F_{cd} \gamma^{ac} \gamma^{bd}} \right] \\ & - q \int d^3 \sigma \varepsilon^{abc} \mathcal{A}_\mu \partial_a X^\mu F_{bc} - \frac{\beta}{3!} \int d^3 \sigma \varepsilon^{abc} \partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \mathcal{A}_{\mu\nu\lambda} \end{aligned} \quad (21)$$

The second Chern-Simmons-like term in (21) is a special case of a class of Chern-Simmons-like couplings of extended objects to external electromagnetic fields proposed in ref.[17].

Let us recall the physical significance of $\mathcal{A}_{\mu\nu\lambda}$ [18]. In $D = 4$ when adding kinetic term for $\mathcal{A}_{\mu\nu\lambda}$ coupled to gravity (see Eq.(40) below), its field-strength:

$$\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]} = \mathcal{F}\sqrt{-G}\varepsilon_{\kappa\lambda\mu\nu} \quad (22)$$

with a single independent component \mathcal{F} produces *dynamical (positive) cosmological constant*:

$$K = 4\pi G_N \mathcal{F}^2 . \quad (23)$$

The constraints (17)–(18) (gauged-fixed equations of motion w.r.t. γ^{ab} remain unaltered for the action (21). Using the same gauge choice ($\gamma^{0i} = 0, \gamma^{00} = -1$) and ansatz for the world-volume gauge field-strength ($F_{0i}(A) = 0$), the equations of motion w.r.t. A_a now acquire the form:

$$\partial_i X^\mu \partial_j X^\nu \mathcal{F}_{\mu\nu}(\mathcal{A}) = 0 \quad , \quad \partial_i \chi + \sqrt{2}q \partial_0 X^\mu \partial_i X^\nu \mathcal{F}_{\mu\nu}(\mathcal{A}) = 0 , \quad (24)$$

(recall $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ – the brane tension, $\mathcal{F}_{\mu\nu}(\mathcal{A}) = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$). Eqs.(24) tell us that consistency of charged WILL-brane dynamics implies that the external space-time Maxwell field must have zero magnetic component normal to the brane, as well as that the projection of the external electric field along the brane must be proportional to the gradient of the brane tension. Finally, the X^μ equations of motion for (21) read:

$$\begin{aligned} & \tilde{\square}^{(3)} X^\mu + (-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl} \partial_k X^\nu \partial_l X^\lambda) \Gamma_{\nu\lambda}^\mu \\ & - q \frac{\gamma^{kl} (\partial_k X \partial_l X)}{\sqrt{2} \chi} \partial_0 X^\nu \mathcal{F}_{\lambda\nu} G^{\lambda\mu} - \frac{\beta}{3!} \varepsilon^{abc} \partial_a X^\kappa \partial_b X^\lambda \partial_c X^\nu G^{\mu\rho} \mathcal{F}_{\rho\kappa\lambda\nu} = 0 , \end{aligned} \quad (25)$$

where $\mathcal{F}_{\rho\kappa\lambda\nu}$ is given as in (22) and:

$$\tilde{\square}^{(3)} \equiv -\frac{1}{\chi \sqrt{\gamma^{(2)}}} \partial_0 \left(\chi \sqrt{\gamma^{(2)}} \partial_0 \right) + \frac{1}{\chi \sqrt{\gamma^{(2)}}} \partial_i \left(\chi \sqrt{\gamma^{(2)}} \gamma^{ij} \partial_j \right) \quad (26)$$

where $\gamma^{(2)} = \det \|\gamma_{ij}\|$ (recall Eqs.(14)–(15)).

As a consequence of of the constraints (17)–(18) and the equations of motion for X^μ (25) we also obtain:

$$\partial_0 (\partial_i X^\mu \partial_j X^\nu) = 0 \quad (27)$$

III. *WILL*-BRANE IN SPHERICALLY-SYMMETRIC BACKGROUNDS

Let us consider the general form of spherically-symmetric gravitational background:

$$(ds)^2 = -A(r,t)(dt)^2 + B(r,t)(dr)^2 + C(r,t)[(d\theta)^2 + \sin^2(\theta)(d\phi)^2] \quad (28)$$

where specifically:

$$A(r) = B^{-1}(r) = 1 - \frac{2G_N M}{r} \quad (29)$$

for Schwarzschild geometry;

$$A(r) = B^{-1}(r) = 1 - \frac{2G_N M}{r} + \frac{G_N Q^2}{r^2} \quad (30)$$

for Reissner-Nordström geometry;

$$A(r) = B^{-1}(r) = 1 - Kr^2 \quad (31)$$

for (anti) de Sitter geometry;

$$A(r) = B^{-1}(r) = 1 - Kr^2 - \frac{2G_N M}{r} \quad (32)$$

for Schwarzschild-(anti)-de-Sitter geometry;

$$A(r) = B^{-1}(r) = 1 - Kr^2 - \frac{2G_N M}{r} + \frac{G_N Q^2}{r^2} \quad (33)$$

for Reissner-Nordström-(anti)-de-Sitter geometry.

We will use the following ansatz:

$$X^0 \equiv t = \tau \quad , \quad X^1 \equiv r = r(\tau, \sigma^1, \sigma^2) \quad , \quad X^2 \equiv \theta = \sigma^1 \quad , \quad X^3 \equiv \phi = \sigma^2 \quad (34)$$

$$\gamma_{ij} = a(\tau) \left((d\sigma^1)^2 + \sin^2(\sigma^1)(d\sigma^2)^2 \right) \quad (35)$$

Substituting (34)–(35) into the *WILL*-brane equations of motion one gets:

- Equations for $r(\tau, \sigma^1, \sigma^2)$ from the lightlike (17) and Virasoro-type (18) constraints:

$$\frac{\partial r}{\partial \tau} = \pm \sqrt{\frac{A}{B}} \quad , \quad \frac{\partial r}{\partial \sigma^i} = 0 \quad (36)$$

- A strong restriction on the gravitational background itself coming from (27):

$$\frac{dC}{d\tau} \equiv \left(\frac{\partial C}{\partial t} \pm \sqrt{\frac{A}{B}} \frac{\partial C}{\partial r} \right) \Big|_{t=\tau, r=r(\tau)} = 0 \quad (37)$$

Eq.(37) tells us that the (squared) sphere radius $R^2 \equiv C(r, t)$ must remain constant along the *WILL*-brane trajectory. For static backgrounds $R^2 \equiv C(r)$ Eqs.(37),(36) imply:

$$r(\tau) = r_0 \text{ (= const) } , \quad A(r_0) = 0 \quad (38)$$

Eq.(38) is of primary importance as it shows that *the WILL-brane automatically positions itself on the event horizon.*

- *WILL*-brane equations of motion (25) for $X^0 \equiv t$ and $X^1 \equiv r$ turn out to be proportional to each other and reduce to an equation for the conformal factor $a(\tau)$ of the internal membrane metric (35) :

$$\partial_\tau(\chi a) + \chi a \frac{\frac{\partial}{\partial t}\sqrt{AB} \pm \partial_r A}{\sqrt{AB}} \pm \chi \frac{\partial_r C}{\sqrt{AB}} \mp \sqrt{2}q \frac{C}{\sqrt{AB}} \mathcal{F}_{0r} \pm \beta \mathcal{F} C \sqrt{AB} = 0 \quad (39)$$

where \mathcal{F} is the independent component of the rank 4 field-strength (22) (here again one sets at the end $t = \tau$, $r = r(\tau)$).

IV. BULK GRAVITY-MATTER COUPLED TO *WILL*-BRANE

A. Action and Equations of Motion

Let us now consider the following coupled Einstein-Maxwell-*WILL*-brane system adding also a coupling to a rank 3 gauge potential:

$$S = \int d^4x \sqrt{-G} \left[\frac{R(G)}{16\pi G_N} - \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - \frac{1}{4!2} \mathcal{F}_{\kappa\lambda\mu\nu} \mathcal{F}^{\kappa\lambda\mu\nu} \right] + S_{\text{WILL-brane}} . \quad (40)$$

Here $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, $\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa} \mathcal{A}_{\lambda\mu\nu]}$ as in (22), and the *WILL*-brane action is the same as in (21):

$$S_{\text{WILL-brane}} = - \int d^3\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab} F_{cd} \gamma^{ac} \gamma^{bd}} \right] - q \int d^3\sigma \varepsilon^{abc} \mathcal{A}_\mu \partial_a X^\mu F_{bc} - \frac{\beta}{3!} \int d^3\sigma \varepsilon^{abc} \partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \mathcal{A}_{\mu\nu\lambda} . \quad (41)$$

The equations of motion for the *WILL*-membrane subsystem are the same as (17)–(18) and (24)–(25), whereas the equations for the space-time fields read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi G_N (T_{\mu\nu}^{(EM)} + T_{\mu\nu}^{(rank-3)} + T_{\mu\nu}^{(brane)}) \quad (42)$$

$$\partial_\nu \left(\sqrt{-G} G^{\mu\kappa} G^{\nu\lambda} \mathcal{F}_{\kappa\lambda} \right) + j^\mu = 0 \quad (43)$$

$$\varepsilon^{\lambda\mu\nu\kappa} \partial_\kappa \mathcal{F} + \beta \int d^3\sigma \delta^{(4)}(x - X(\sigma)) \varepsilon^{abc} \partial_a X^\lambda \partial_a X^\mu \partial_a X^\nu = 0 \quad (44)$$

where in the last equation we have used relation (22). The explicit form of the energy-momentum tensors read:

$$T_{\mu\nu}^{(EM)} = \mathcal{F}_{\mu\kappa} \mathcal{F}_{\nu\lambda} G^{\kappa\lambda} - G_{\mu\nu} \frac{1}{4} \mathcal{F}_{\rho\kappa} \mathcal{F}_{\sigma\lambda} G^{\rho\sigma} G^{\kappa\lambda} , \quad (45)$$

$$T_{\mu\nu}^{(rank-3)} = \frac{1}{3!} \left[\mathcal{F}_{\mu\kappa\lambda\rho} \mathcal{F}_{\nu}{}^{\kappa\lambda\rho} - \frac{1}{8} G_{\mu\nu} \mathcal{F}_{\kappa\lambda\rho\sigma} \mathcal{F}^{\kappa\lambda\rho\sigma} \right] = -\frac{1}{2} \mathcal{F}^2 G_{\mu\nu} , \quad (46)$$

$$T_{\mu\nu}^{(brane)} = -G_{\mu\kappa} G_{\nu\lambda} \int d^3\sigma \frac{\delta^{(4)}(x - X(\sigma))}{\sqrt{-G}} \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\kappa \partial_b X^\lambda , \quad (47)$$

(recall $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ – the brane tension, cf.(5)). The charge current in (43) produced by the *WILL*-brane is given by:

$$j^\mu = q \int d^3\sigma \delta^{(4)}(x - X(\sigma)) \varepsilon^{abc} F_{bc} \partial_a X^\mu . \quad (48)$$

B. Static Spherically Symmetric Solutions. Matching Across Horizon

We find the following static spherically symmetric solutions for the bulk gravity-matter system coupled to a charged *WILL*-brane (40). The bulk space-time consists of two regions separated by the *WILL*-brane as a common horizon materialized by the *WILL*-brane:

$$(ds)^2 = -A_{(\mp)}(r)(dt)^2 + \frac{1}{A_{(\mp)}(r)}(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)(d\phi)^2] \quad (49)$$

where the subscript $(-)$ refers to the region inside, whereas the subscript $(+)$ refers to the region outside the horizon at $r = r_0 \equiv r_{\text{horizon}}$ with $A_{(\mp)}(r_0) = 0$. We have Schwarzschild-de-Sitter space-time inside horizon:

$$A(r) \equiv A_{(-)}(r) = 1 - K_{(-)} r^2 - \frac{2G_N M_{(-)}}{r} , \quad \text{for } r < r_0 , \quad (50)$$

and Reissner-Norström-de-Sitter space-time outside horizon:

$$A(r) \equiv A_{(+)}(r) = 1 - K_{(+)} r^2 - \frac{2G_N M_{(+)}}{r} + \frac{G_N Q^2}{r^2} , \quad \text{for } r > r_0 , \quad (51)$$

with Reissner-Norström (squared) charge given by $Q^2 = 8\pi q^2 r_0^4$. The dynamically induced (due to the presence of the rank 3 tensor gauge potential) cosmological constant is different inside and outside the horizon:

$$K_{(\pm)} = \frac{4}{3}\pi G_N \mathcal{F}_{(\pm)}^2 \quad \text{for } r \geq r_0 \quad (r \leq r_0) \quad , \quad \mathcal{F}_{(+)} = \mathcal{F}_{(-)} - \beta \quad , \quad (52)$$

where $\mathcal{F}_{(\pm)}$ are the corresponding constant values of the rank 4 tensor field-strength (22) according to Eq.(44).

As already discussed in Section III (cf. Eqs.(36)–(38)), the *WILL*-membrane locates itself automatically on (“straddles”) the common event horizon at $r = r_0$:

$$X^0 \equiv t = \tau \quad , \quad X^2 \equiv \theta = \sigma^1 \quad , \quad X^3 \equiv \phi = \sigma^2$$

$$X^1 \equiv r(\tau, \sigma^1, \sigma^2) = r_0 = \text{const} \quad \text{where } A_{(-)}(r_0) = A_{(+)}(r_0) = 0 \quad (53)$$

Besides inducing the jump (52) in the space-varying cosmological constant, the *WILL*-brane also causes the following discontinuity in the normal derivative of the metric component $A(r)$ resulting from the *WILL*-brane contribution (47) to the r.h.s. of Einstein Eqs.(42):

$$\partial_r A_{(+)} \Big|_{r=r_0} - \partial_r A_{(-)} \Big|_{r=r_0} = -16\pi G_N \chi \quad . \quad (54)$$

Eq.(54) is easily obtained upon using the simple expressions for the components of the Ricci tensor corresponding to the metric (28) $R_0^0 = R_1^1 = -\frac{1}{2r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial}{\partial r} A(r))$ [19] and upon substituting (53) in (47).

For the Maxwell subsystem (43),(48) we have Coulomb field outside horizon generated by the surface charge of the *WILL*-brane:

$$\mathcal{A}_0 = \frac{\sqrt{2} q r_0^2}{r} \quad , \quad \text{for } r \geq r_0$$

and no electric field inside horizon:

$$\mathcal{A}_0 = \sqrt{2} q r_0 = \text{const} \quad , \quad \text{for } r \leq r_0 \quad ,$$

Apart from the matching conditions for the metric components (54) and the induced cosmological constant (52) when crossing the *WILL*-brane hypersurface, *WILL*-brane dynamics imposes an important third matching condition. Namely, the remaining non-trivial

WILL-brane Eq.(39) reduces now to the following two equations because of the two different space-time geometries inside and outside the horizon:

$$\partial_0(\chi a) + \chi a \partial_r A_{(-)} \Big|_{r=r_0} - (2\chi r_0 + \beta \mathcal{F}_{(-)} r_0^2) = 0 , \quad (55)$$

$$\partial_0(\chi a) + \chi a \partial_r A_{(+)} \Big|_{r=r_0} - (2\chi r_0 + \beta \mathcal{F}_{(+)} r_0^2 - 2q^2 r_0^2) = 0 . \quad (56)$$

Eqs.(55),(56) should be yield the same solution for the conformal factor $a(\tau)$ of the internal brane metric, *i.e.*, we have the following additional matching condition:

$$\partial_r A_{(+)} \Big|_{r=r_0} - \partial_r A_{(-)} \Big|_{r=r_0} = - \frac{r_0(2q^2 + \beta^2) \partial_r A_{(-)} \Big|_{r=r_0}}{2\chi + \beta r_0 \mathcal{F}_{(-)}} \quad (57)$$

The matching conditions (54) and (57) allow to express all physical parameters of our solution in terms of 3 free parameters (q, β, \mathcal{F}) where:

- (a) q – *WILL*-brane surface electric charge density;
- (b) β – *WILL*-brane (Kalb-Rammond-type) charge w.r.t. rank 3 space-time gauge potential $\mathcal{A}_{\lambda\mu\nu}$;
- (c) $\mathcal{F}_{(-)}$ – vacuum expectation value of $\mathcal{F}_{\kappa\lambda\mu\nu}$ (22) in the interior region.

The corresponding explicit expressions read:

- Horizon radius:

$$r_0^2 = \frac{1}{4\pi G_N \left(\mathcal{F}_{(-)}^2 - \beta \mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2} \right)} . \quad (58)$$

- Schwarzschild mass:

$$M_{(-)} = \frac{r_0 \left(\frac{2}{3} \mathcal{F}_{(-)}^2 - \beta \mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2} \right)}{2G_N \left(\mathcal{F}_{(-)}^2 - \beta \mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2} \right)} . \quad (59)$$

- Reissner-Nordström mass:

$$M_{(+)} = M_{(-)} + \frac{r_0}{2G_N \left(\mathcal{F}_{(-)}^2 - \beta \mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2} \right)} \left(2q^2 + \frac{2}{3} \beta \mathcal{F}_{(-)} - \frac{1}{3} \beta^2 \right) . \quad (60)$$

- Reissner-Nordström charge:

$$Q^2 = 8\pi q^2 r_0^4 . \quad (61)$$

- Space-varying dynamically induced cosmological constant:

$$K_{(\pm)} = \frac{4}{3}\pi G_N \mathcal{F}_{(\pm)}^2 \quad , \quad \mathcal{F}_{(+)} = \mathcal{F}_{(-)} - \beta . \quad (62)$$

- Brane tension:

$$\chi = \frac{r_0}{2} \left(q^2 + \frac{\beta^2}{2} - 2\beta \mathcal{F}_{(-)} \right) . \quad (63)$$

To determine the type of the common horizon materialized by the *WILL*-brane, we need the expressions for the slopes of the metric coefficients $A_{(\pm)}(r)$ at $r = r_0$. Using expressions (58)–(63) we find:

$$\partial_r A_{(+)} \Big|_{r=r_0} = -\partial_r A_{(-)} \Big|_{r=r_0} , \quad (64)$$

$$\partial_r A_{(-)} \Big|_{r=r_0} = 8\pi G_N \chi = 4\pi G_N r_0 \left(q^2 + \frac{\beta^2}{2} - 2\beta \mathcal{F}_{(-)} \right) . \quad (65)$$

Therefore, in view of Eqs.(64)–(65) (henceforth for definiteness we assume $\beta > 0$ [22]) :

- (i) In the area of parameter space $\mathcal{F}_{(-)} > \frac{q^2 + \frac{\beta^2}{2}}{2\beta}$ (i.e., when $\chi < 0$ – negative brane tension):

(a) the common horizon is the De-Sitter horizon from the point of view of the interior Schwarzschild-de-Sitter geometry;

(b) the common horizon is the external Reissner-Nordström horizon (the larger one) from the point of view of the exterior Reissner-Nordström-de-Sitter geometry.

The typical form of $A(r)$ is shown in Fig.1.

- (ii) In the opposite area of parameter space $\mathcal{F}_{(-)} < \frac{q^2 + \frac{\beta^2}{2}}{2\beta}$ (i.e., when $\chi > 0$ – positive brane tension):

(a) the common horizon is the Schwarzschild horizon from the point of view of the internal Schwarzschild-de-Sitter geometry;

(b) the common horizon is internal (the smaller one) Reissner-Nordström horizon from the point of view of the external Reissner-Nordström-de-Sitter geometry.

Remark. In the area of parameter space (i), although the brane tension χ is negative, the brane can be stable due to negative pressure caused by the difference between the “inside” and “outside” dynamically induced cosmological constants $\mathcal{F}_{(-)} > \mathcal{F}_{(+)}$ (cf. Eq.(62)).

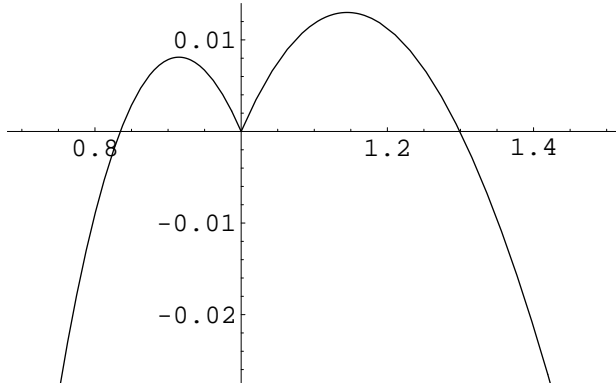


FIG. 1: Shape of $A(r)$ as a function of the dimensionless ratio $x \equiv r/r_0$

In the parameter area (i), let us take the particular case $q = 0$ and $\mathcal{F}_{(-)} = \beta$, *i.e.*, no electromagnetic interactions present, vanishing cosmological constant in the exterior region and the *WILL*-brane “sits” on the common horizon of an interior Schwarzschild-de-Sitter region matched with an exterior pure Schwarzschild region. The corresponding parameters are given as:

$$A_{(-)}(r) = 1 - K_{(-)}r^2 - \frac{2G_N M_{(-)}}{r} \quad , \quad A_{(+)}(r) = 1 - \frac{2G_N M_{(+)}}{r} \quad , \quad (66)$$

for $r < r_0$ and $r > r_0$, respectively, where:

$$r_0 = \frac{1}{\sqrt{2\pi G_N}} \frac{1}{\beta} \quad , \quad K_{(-)} = \frac{4}{3}\pi G_N \beta^2 \quad , \quad M_{(+)} = 3M_{(-)} = \frac{1}{\sqrt{2\pi G_N}} \frac{1}{2\beta G_N} \quad . \quad (67)$$

In this special case the brane tension becomes $\chi = -\frac{1}{\sqrt{2\pi G_N}} \frac{3}{4}\beta$.

Let us also mention the simple special case $\beta = \mathcal{F}_{(-)} = 0$, *i.e.*, matching interior purely Schwarzschild black hole region with exterior purely Reissner-Nordström black hole region. This case has already been discussed in [11].

The results in the present Section show that the Einstein-Maxwell-*WILL*-brane system (40) can be viewed as the first explicit dynamical realization of the “membrane paradigm” in black hole physics [6]. Indeed, the brane dynamics determined by the action (41) dictates that the brane must be inherently lightlike and that it necessarily has to locate itself on a black hole event horizon. Moreover, its properties dynamically determine the nature of the surrounding bulk gravity and matter.

V. TRAPPING POTENTIAL WELL AROUND COMMON HORIZON

Consider planar motion of a (charged) test particle with mass m and electric charge q_0 in a gravitational background given by the solutions in Section IV, namely, internal Schwarzschild-de-Sitter region matched with external Reissner-Norström-de-Sitter region along a common event horizon materialized by Weyl-conformally invariant lightlike brane, which simultaneously serves as material and charge source for gravity and electromagnetism. Conservation of energy yields (E, J – energy and orbital momentum of the test particle; prime indicates proper-time derivative):

$$\frac{E^2}{m^2} = r'^2 + V_{eff}^2(r)$$

$$V_{eff}^2(r) = A_{(-)}(r) \left(1 + \frac{J^2}{m^2 r^2} \right) + \frac{2E q_0}{m^2} \sqrt{2} q r_0 - \frac{q_0^2}{m^2} 2q^2 r_0^2 \quad (r \leq r_0)$$

$$V_{eff}^2(r) = A_{(+)}(r) \left(1 + \frac{J^2}{m^2 r^2} \right) + \frac{2E q_0}{m^2} \frac{\sqrt{2} q r_0^2}{r} - \frac{q_0^2}{m^2} \frac{2q^2 r_0^4}{r^2} \quad (r \geq r_0) \quad (68)$$

where $A_{(\mp)}$ are the same as in (50) and (51).

Taking into account (64)–(65) we see that in the parameter interval:

$$\mathcal{F}_{(-)} \in \left(\frac{q^2 + \frac{\beta^2}{2}}{\beta}, \infty \right) \quad (69)$$

the (squared) effective potential $V_{eff}^2(r)$ acquires a potential “well” in the vicinity of the *WILL*-brane (the common horizon) with a minimum on the brane itself.

Let us illustrate graphically the simplest physically interesting case with $q = 0$, $\mathcal{F}_{(-)} = \beta$ and β – arbitrary, i.e., matching of Schwarzschild-de-Sitter interior (with dynamically

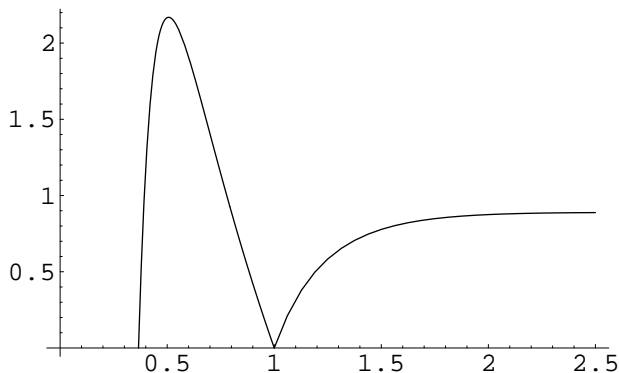


FIG. 2: Shape of $V_{eff}^2(r)$ as a function of the dimensionless ratio $x \equiv r/r_0$

generated cosmological constant) against pure Schwarzschild exterior (with *no* cosmological constant) along the *WILL*-brane as their common horizon (cf. Eqs.(66)–(67)). The typical form of $V_{eff}^2(r)$ is shown in Fig.2.

Thus, we conclude that if a test particle moving towards the common event horizon loses energy (*e.g.*, by radiation), it may fall and be trapped by the potential well, so that it neither falls into the black hole nor can escape back to infinity. In this way one could form a “cloud” of trapped particles around the horizon.

VI. CONCLUSIONS AND OUTLOOK

The present paper as well as the previous ones [10, 11] show that modifying of world-sheet (world-volume) integration measure (cf. Eq.(3)) significantly affects string and p -brane dynamics.

- Acceptable dynamics in the new class of string/brane models (2) *naturally* requires the introduction of auxiliary world-sheet/world-volume gauge fields.
- By employing square-root Yang-Mills actions for the auxiliary world-sheet/world-volume gauge fields one achieves *Weyl conformal symmetry* in the new class of p -brane theories *for any* p .
- Here the string/brane tension is *not* a constant scale given *ad hoc*, but rather an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.
- Weyl-conformally invariant p -brane models (2) describe *intrinsically lightlike* p -branes for any even p (acronym – *WILL*-branes) with no quantum conformal anomalies ($(p + 1) = \text{odd}$).
- When put in a gravitational black hole background, the *WILL*-brane automatically positions itself on (“materializes”) the event horizon.
- Coupled Einstein-Maxwell-*WILL*-membrane system possesses a self-consistent solution where the *WILL*-membrane serves as a source for gravity and electromagnetism. Moreover, it generates dynamical cosmological constant and automatically “straddles”

the common event horizon for a Schwarzschild-de-Sitter space-time region (in the interior) and Reissner-Nordström-de-Sitter space-time region (in the exterior). This model is the first explicit dynamical realization of the “membrane paradigm” in black hole physics.

- It is very reasonable that negative surface tension on the horizon, produced by the *WILL*-brane “sitting” on it, is the cause of a stable configuration where the positive cosmological constant inside the horizon is larger than the one outside the horizon (once again due to the presence of the *WILL*-brane). The space-varying cosmological constant causes a pressure difference that gives rise to an inward force on the horizon and this is compensated by the negative surface tension due to the *WILL*-brane, which pushes the horizon surface outwards.
- The non-trivial matching of different black hole regions along the common horizon “straddled” by the *WILL*-brane produces a *trapping potential* “well” for infalling test particles in the vicinity of the common horizon “guarding” the inner Schwarzschild-like horizon of the interior Schwarzschild-de-Sitter region.

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which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

- [21] “Square-root” Maxwell (Yang-Mills) action in $D = 4$ was originally introduced in the first ref.[16] and later generalized to “square-root” actions of higher-rank antisymmetric tensor gauge fields in $D \geq 4$ in the second and third refs.[16].
- [22] Taking $\beta < 0$ does not qualitatively change the results below. Indeed, in all relevant expressions (58)–(60) and (63) β appears always multiplied by $\mathcal{F}_{(-)}$. Therefore, changing the sign of β can always be compensated by changing the sign of the 4-index field strength which is an obvious symmetry of the action (40).